

Resource Guide to Mathematics Curriculum Materials for High Ability Learners, Grades K – 8

Center for Gifted Education
The College of William and Mary
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Introduction

Mathematics education has undergone many changes over the past few decades. Those changes are reflected in the curriculum materials that are currently available. This guide is intended to help the teachers, administrators, and parents of high ability mathematics students explore the materials on the market today. The guide gives a set of criteria for judging new materials and gives examples of different types of materials that meet those criteria.

Mathematics and Teaching -- The State of the Art

In many classrooms, the teaching and learning of mathematics has changed dramatically over the last decade. A focus on "the what and the why of mathematics" has replaced an emphasis on computational skills and rote algorithms. Discussion, writing, and manipulative materials have taken the place of "ditto" worksheets. Problem solving, reasoning, and real world applications have become essential in the classroom experience. Calculators and computers have become an integral part of learning. "Consumer math" is emphasized less and algebra is encouraged for all students.

The National Council of Teachers of Mathematics (NCTM) has taken the lead in this area by writing its *Curriculum and Evaluation Standards* (1989) and its update, *Principles and Standards for School Mathematics* (2000). It is not a how-to guide but reflects the goals and principles that should guide and improve math programs at all levels. The document sets the tone for mathematics education in this country. In addition to content strands Geometry, Algebra, Number and Operations, Measurement, Data Analysis and Probability, the *Standards* place major emphasis on these broad areas of mathematics: Problem Solving, Communication, Reasoning and Proof, Connections and Representation. There is a movement away from the traditional emphases on computation and rote math activities. Group problem solving and hands-on activities are the mode in the recommendations for mathematics curriculum. Mathematics is seen as the foundational discipline for other disciplines and therefore connections to other content areas and real-world problems become necessary in the teaching of mathematics. Mathematical models and simulations as applied to many disciplines need to be included. Recommendations in the Standards suggest that activities should grow out of problem situations and that knowledge should emerge from experience with problems. Various representations of data or problem situations are used to organize and express mathematical ideas or create models.

NCTM Standards strongly encourages appropriate use of technology such as calculators, software, and internet resources. The effect of this technological emphasis in teaching and

learning has been to change the very nature of the problems that the students are asked to solve. The methods used to solve problems are diverse. No longer should we expect an answer from students without discussion of how the problem was solved. No longer must the solution be the same for each student.

Implications of the Standards for High Ability Learners

Access to the curriculum proposed in the *Standards* is offered to all students. All students should experience the same subject matter but in different ways. So what does this mean for high ability learners? There are many differences in the depth, breadth, pacing, level, levels of complexity of content and curriculum materials that are suitable for students at the high end of the continuum of interests and abilities. *Standards* documents and supporting resources provide many excellent examples of problems and activities that would be challenging for strong mathematics students. Suitable applications for high ability learners need to be identified. But how do we know them when we see them?

This guide is meant to assist in the search for existing materials that meet the high end of the expectations of *Standards* and the needs of high ability students. Criteria will be outlined for teachers to use when making curriculum selection decisions (at the textbook/program level or the daily lesson level) and some sample materials will be referenced.

Curriculum Materials

Textbooks have traditionally driven mathematics curriculum in the United States. Often teachers have worked their way through the teacher's guide beginning with Chapter 1, but do not get to the end of the book. A great deal of the content is repeated from one grade level text to the next. In a study of three commonly used basal textbook series, Flanders (1987) found from 28% to 70% of the mathematics was repetition from one grade to the next.

Most publishers design texts to appeal to large markets and textbook adoption committees in states such as California, Texas, and Florida. Since textbook publishers want to appeal to everyone and offend no one, the product that they market is not usually radical in any respect and is aimed at the average student. Often extra workbooks are sold containing remedial worksheets or challenge problems. However, an extra workbook is not sufficient to meet the needs of highly able mathematics students. They need curriculum that moves at a faster pace, is less repetitive, goes deeper into important thought processes, and covers a broader set of ideas than that encountered in a commonly used textbook series. It is difficult to accomplish all of this with a given basal text, especially since the needs of the mathematically talented can be highly variable. Even the new versions of "Standards-based" texts are typically not flexible enough to encompass all the needs of high ability students. Mathematically talented students who are not exposed to a rich curriculum risk boredom and may consequently desire to discontinue their study of mathematics prematurely. Table I gives sample Standards and corresponding adaptations that are necessary for high ability learners.

It is difficult to meet the needs of gifted mathematics students by using a textbook alone. While a textbook does contain basic information that must be learned by all students, it should not be the only source of information for mathematically talented students. Challenges of a more demanding nature are necessary for these students. The following are general suggestions for addressing the needs of the mathematically talented through basic textbooks:

1. Accelerate most gifted students by at least one grade level in the textbook series being used. Handle greater acceleration for truly exceptional students on a case by case basis.
2. Give a pre-assessment to students so that they can be started at a place in the text that is appropriate for most of them. If the teacher does not start at the beginning, it is more likely that the chapters at the end of the book will be reached. These chapters contain material that is more likely to be new and challenging to the students. This approach lessens the likelihood of lingering on topics that are repetition from previous years.
3. Raise the challenge level of textbook activities by assigning the end of a problem set (presumably the harder exercises) rather than the beginning. Assign extension or "challenge" problems from the text.
4. Use the textbook as a support rather than the only source of curriculum. Look at the suggestions in this guide for materials that will help enrich and extend the textbook topics or enrich the classroom with new ideas that are not included in the text.

The Development of This Guide

In an attempt to identify the kinds of materials that would be suitable for high ability mathematics students and to locate examples, staff members of the Center for Gifted Education undertook a project in which they identified criteria by which mathematics curriculum materials could be judged and then located examples of such exemplary materials.

This guide is the result of that process. It is intended to help teachers, program administrators, and parents (who often provide an enrichment component for their children) locate and select materials for their high ability students that will appropriately enrich the study of mathematics. Since textbooks cannot be considered the sole source of curriculum, the use of this Resource Guide will facilitate the otherwise daunting search process. Users of this guide may use the annotated lists of recommended materials that appear in the guide to begin making resource decisions. They may also use the criteria review sheets contained in the guide to evaluate the utility of other materials.

There are many mathematics curriculum materials on the market today. No single selection will work for all students; a set of resources is more appropriate. We hope that this guide will both institute and sharpen the search for materials which constitute a good match for students.

References

- Flanders, J. R. (1987). How much of the content in mathematics textbooks is new? *Arithmetic Teacher*, 35, 18-23.
- National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards*. Reston, VA: Author.
- National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Mathematics Curriculum Evaluation Criteria

The following criteria were used in evaluating mathematics materials. Each criterion is accompanied by a description.

Users of this guide are encouraged to employ the criteria in evaluating other mathematics materials not found in this guide. It is unlikely that any single material will satisfy all elements in these lists. Part I lists features that one would expect to see in a comprehensive text in nearly any subject area. Therefore, many of these elements may not be observed in a specialized or unique publication of a supplementary nature. Part II addresses mathematical content and process. Part III looks specifically at the needs of the high ability learner.

Criteria

Use the following rating scale to rate the criteria as observed in the unit:

- 0 -- Not observed
- 1 -- Minimally present
- 2 -- Somewhat emphasized
- 3 -- Exemplary Use

Part I: General Curriculum Design Features

Rationale and Purpose

This feature addresses the reasons for developing a particular unit of study for use with a given group of learners at a particular stage of development. It provides the reader with a sense of the importance of the topic under study and why it is being taught.

- Clear and understandable
Teachers can easily tell why the selected content is important. Rationale and purpose are written in meaningful and clear language rather than in educational and/or mathematical jargon.
- Substantive
Content is important and valuable; publishers/authors provide reasons why content was selected or the value is self-evident.

Lesson objectives

This feature provides the focus and direction for learning in the unit. It specifies anticipated outcomes for students as a result of their being taught a lesson.

- Clear and understandable
A curriculum reader would know immediately what the students will be able to know and/or be able to do as a result of a lesson.

- Measurable

Techniques or suggestions are provided for teachers so they can tell whether or not they have achieved the learning outcomes.

- Supportive of unit rationale & purpose

The goals of an individual lesson are consistent with the goals of the curriculum as a whole. There is a clear sense of building toward unit goals.

Activities

This feature specifies what teachers will do to facilitate learning and what students will do to learn.

- Used to explore, discover, clarify, and extend content (not just drill)

Activities should go beyond drill and practice of skills. Exploration and thinking should be embedded in the activities.

- Includes interdisciplinary material

Other content areas such as language arts and science are integrated into the activities in a way that is natural rather than contrived for the sake of claiming connections.

- High active to passive ratio

Students are engaged in dynamic activities rather than teachers explaining and students doing worksheets. Where appropriate, manipulatives should be used.

- Minimal drill and practice

Even though some practice is necessary, it should not be the dominant mode of student activity.

- Students have opportunities to work both independently and in groups

The material suggests both large and small group activities and different ways these groups can be formed and used.

- Open-ended exploration

Students are given opportunities to "mess around" with ideas and materials without a preconception of what it is that they will discover.

Instructional strategies

This feature provides direction to readers about the major approaches to teaching that will be undertaken. It specifies heuristic models, questioning techniques, and conferencing approaches used by the teacher.

- Varied

Several different forms of instruction are suggested (e.g., inquiry activities, lecture, independent work, etc.)

- Opportunities for open inquiry

Problems, projects, etc. are open-ended with respect to the solution or approach to the solution. Students are encouraged to formulate questions and explore possible answers to those questions.

- Use of various types of questions (e.g., convergent, divergent, evaluative)

There are a variety of questions suggested which stimulate different levels of thought, from knowledge through evaluation.

- Opportunity for student reflection and evaluation of their own thinking

Students are engaged in reflection through discussion or writing and are asked questions such as What did you learn from that? Was your hypothesis correct? How could you do that better?

Assessment procedures

These are the various ways that teachers get feedback about what students know and can do.

- Unit pre-post assessment procedures

There are opportunities at both the beginning and end of the unit to measure knowledge so that relative gain can be measured.

- Use of multiple means of assessment including observation

Teachers are encouraged to collect many types of data that will reflect the level of student achievement and understanding. These include write-ups of problem solutions, projects, etc.

- Use of authentic modes of assessment

The assessments are applications of the learning that has occurred in the unit in a context similar to the learning environment. There are opportunities to evaluate students doing mathematical tasks, not just written tests.

- Students are acknowledged for good reasoning (process assessment)

Students are asked to explain their thinking, provide reasons for their solutions, or describe the process they used to arrive at a solution. Even if the answer is not correct, appropriate forms of reasoning are encouraged and rewarded.

- Use of appropriate scoring rubrics

Explanations of rubrics and their applications are provided. Students should know and be able to apply rubrics to their own works.

- Use of self-assessments

Students are asked to evaluate their own performance and level of understanding.

Materials and Resources

- Bibliography for student extensions

Lists of materials are provided to guide extensions for students.

- Supportive bibliography for teachers
Resources are suggested for teachers who wish to enhance their knowledge of the subject.

- Handout material contributes to enhancement of learning (not busy work)
Handouts contain food for thought, not just drill and practice.

Extension ideas

- Worthwhile, related activities for students to pursue independently
Follow-up activities are provided that go beyond the standard prepared unit of study but are logical next activities and are based on unit work. These include ideas, activities, and resources. Suggestions for both group and independent work should be encouraged.

Technology (especially calculators and computers)

- Actively engages students in higher order thinking skills and activities
Calculators are used to facilitate problem solving that would not be possible without the aid of such technology. For example: How long would it take to count to a billion? Computers or graphing calculators might be used to write a program that could be used to solve a problem.
- Enhances and complements instruction
The technology allows students to see relationships that are difficult to demonstrate without it. It allows for a shift in emphasis toward problem solving rather than computation as the focus of "word problems." It should not be merely a novel means of drill and practice. Some appropriate applications are:
 - *Demonstration of mathematical principles interactively such as the relationship of radius to circumference of a circle.*
 - *Use of a calculator to check a difficult calculation.*
 - *Acquisition of information that is not readily available elsewhere such as the value of $\tan(2/3)$.*

Part II: Mathematics Content and Process

Content

- Content is organized around major mathematical ideas.
For example, functions, inverses, patterns, and problem solving rather than skills.
- Mathematical concepts are covered in depth.
The treatment of concepts goes beyond minimum exposure. For example, place value treated beyond base ten.
- History of mathematical ideas, concepts, mathematicians is included.
This is sometimes found in sidebars and extensions. Preferably incorporated into problems.

- Mathematics is accurate.

There should be no mistakes in the presentation of material and the terminology should be appropriate.

- Mathematics is presented understandably for students.

The style of presentation in written materials is readable for students; sufficient examples and graphic aids are provided to make the ideas clear.

- Material progresses from concrete to abstract.

New concepts are supported with concrete manipulatives, diagrams, pictures, analogies, examples, etc. before moving into abstractions.

Process

- Problem solving is an integral part of the curriculum.

Problems are not just added at the end of the lesson or unit. Problems are not concocted for application of skill but rather are major problems that provide the context for concept development or skill-building.

- Students are required to communicate ideas.

- In discussion with each other, the teacher, or other people

It should not be at the "What did you get?" level but should be, for example, discussion of strategies, evaluation of solutions, etc.

- Through oral presentations

Students should be expected to find out things and report them to the class. This should not just be fact finding reports but also explanations of problems that the student has solved.

- In written form

Questions are integrated into the curriculum that require students to write about their understandings of ideas, their strategies for solving a problem or their reasoning. Students might be asked to keep a journal that goes beyond accumulating problems.

- In visual form

Students may be asked to create a product with a visual component. It might be a poster, graph, diagram, etc.

- Reasoning is integral to mathematics lessons.

- Students discover ideas and concepts more often than they are told them.

Activities require students to figure things out.

- Students learn why things work as they do.

The workings of ideas are not just given as rules but are explained or discovered in such a way that students know why the ideas work and can accurately explain them to someone

else. Formal proofs would be appropriate for high school. Informal proofs are expected at all levels.

- Opportunities are available for both inductive and deductive reasoning.
 - *Inductive reasoning is the conjecturing of a generalization from specific instances. (A student who measures angles in a few triangles and then conjectures that the sum of the measures of the angles in any triangle is 180° is using inductive reasoning.)*
 - *Deductive reasoning is the process of using logical principles to prove that one statement follows from others. (A student who uses the formula for the area of a rectangle to develop a formula for the area of a right triangle is using deductive reasoning.)*

- Relevant mathematical connections are made.

- To the real world

Students are shown where the math concepts are used outside of the math classroom, to solve problems encountered in daily living, on the job, in a hobby, etc.

- To other areas of mathematics

Links should be made between areas such as measurement and geometry or number theory and algebra. Many areas of the Standards can be addressed within a single lesson.

- To other disciplines

Connections are made to the sciences (temperature, $\text{Distance} = \text{Rate} \times \text{time}$), social studies (map skills, latitude and longitude), art, architecture, language arts, etc.

- Multiple representations are used.

Students are expected to represent problems and results in more than one way. Diagrams, verbal descriptions, tables, graphs, or physical models might be used.

- Students are asked to make conjectures and attempt to verify or prove them.

Students are asked to make educated guesses about how they think things work. Questions such as:

- *What do you think will happen?*
- *See if you can determine a formula.*
- *Find a pattern.*
- *Is it always true? How can you find out?*
- *Find the next number in the sequence. How do you know? Then find the next one and check with other group members.*

- The curriculum illustrates habits of mind of mathematicians (e.g., curiosity, tenacity, collaboration, skepticism).

Situations are provided that stimulate curiosity in students, that make them wonder why things work the way they do..., what would happen if..., try taking it one step further ...

Problems need to be sufficiently challenging that students have to work hard and maybe long to get a result.

- Divergent thinking is encouraged and the thinking process is valued as well as knowledge. *Students are not expected to solve problems in only one way. Some problems have multiple possibilities for a solution.*

- Opportunities are provided for students to work together as well as alone in solving problems.

Both group work and individual work are encouraged.

- Students are encouraged to try out their own ideas (regardless of the likelihood of their success)

Students are asked to make conjectures and then test them rather than just verify what has already been stated as truth.

Part III: Essential Features for High Ability Learners

- The materials contain a high level of sophistication of ideas.

Concepts are addressed that are beyond current skill level and require students to think about important mathematical ideas.

- Opportunities for extensions that challenge the most able learners are present.

Activities or suggestions for activities are suggested that provide a challenging experience for students who need to go beyond the level of the unit activities.

- Potential exists for tailoring content to individual needs of high ability learners

This may be done by coding problems by difficulty level. Some chapters may be designated as more demanding. Extensions may be suggested for independent or small group work.

- Use of higher order thinking skills is integral to the lesson (i.e., analysis, synthesis, and evaluation).

This element is critical in curriculum for high ability learners. Even though there is room for basic facts, these should not be a focus of the material. Must students think in order to participate in the activities or do they just model what they do after examples?

- Materials include key themes.

Major themes may be the framing ideas in the curriculum or they may be integrated into the materials. (e.g., problem solving, rates, change, scale, models)

- Materials have less emphasis on basic skills once mastery level has been demonstrated.

The purpose of the materials is to go beyond the skill level.

- A sufficiently high level of abstraction is encouraged.

Students are asked to go beyond the concrete stages. Variables are used to represent quantities; symbols are used to represent operations and ideas.

- Opportunities for student exploration based on interest are included.

The curriculum provides ideas for extension activities to be chosen by students in areas that are of interest to them.

- Opportunities are provided to create products that are:

- open-ended

The task demand for a product will not produce the same outcome from all students.

Students might be able to design their own product.

- advanced

The product is challenging beyond what most students in the grade level would be expected to do.

Websites

Websites are not formal curriculum. Some of the websites listed in this guide are of interest to teachers as a source of ideas for classroom instruction and activities. However, many are accessible and interesting for students to explore independently. Once you get into one of these sites, you will discover many other hot links to other great resources. A trip to the web will not be the same experience for any two people nor will it be the same each time you visit. Everything on the web is subject to change and some changes occur frequently.

Criteria for Selection

Why are these sites appropriate for the mathematically gifted? We will examine this question by looking at some of the criteria for appropriateness for high ability learners that were identified above.

- The materials contain a high level of sophistication of ideas.
Topics such as infinity are explored.
- Opportunities for extensions that challenge the most able learners are present.
Gifted learners use up resources faster than other students. There are countless challenging problems and ideas for lessons on the web.
- Potential exists for tailoring content to individual needs of high ability learners.
There is something for many different levels, interests, and backgrounds. Age is not a gatekeeper for activities, projects, and problems.
- Use of higher order thinking skills.
There are a number of projects and problems that promote reasoning and problem solving.
- Materials have less emphasis on basic skills once mastery level has been demonstrated.
Most websites go beyond what textbooks include. They do not duplicate textbook-style activities.
- Opportunities for student exploration based on interest are included.
If students have access to the web directly, they can find a wealth of rich topics and problems for exploration.

The following are among the most comprehensive and useful websites that we have encountered. They include something for all levels of students. Many will link you to other sites that we have not listed here. Enjoy your adventure!

Math Forum

<http://mathforum.org>

This site is loaded with information for both students and teachers. Under the Student Center there are Interactive Forum Projects and Links for Students. You can also find a problem of the week under the Projects title which could be used by teachers or students. Other titles include Math Tips and Tricks, Beat Calc, Divisibility Rules, and Multiplication Tips. Under the Link for Students check out the elementary and middle school student centers. The Teacher's Place is an excellent resource for K-12 teachers with everything from issues in math education to Internet math projects. An especially good place to check out is Steve's Dump, which has dozens of math resources for the teacher "Ask Dr. Math" is a great resource for students (or teachers) who have questions. Dr. Math responds to questions at all levels by e-mail and responses are accessible in archives on the site.

Mega Math

<http://www.c3.lanl.gov/mega-math>

This site is a good teacher resource for inspiration at many levels of complexity. Each lesson includes activities, vocabulary, background, big ideas and concepts, evaluation, prep and materials, NCTM, and further study. Projects include coloring a map, knots, graphs, algorithms, and infinity.

Eisenhower National Clearing House for Math and Science

<http://enc.org/lesmath.html>

This site is useful to both teachers and students alike. It includes successful lesson plans and student activities.

K-12 Resources for Mathematics Education

<http://archives.math.utk.edu/k12.html>

This website is mostly useful for teachers. This site includes lesson plans from all different states and specialty schools.

Bob Wilcox's Mathematics Page

<http://geocities.com/Athens/3352>

This link includes a problem of the week and links to sites all over the web.

Directory of Mathematics Links

<http://www.math.fsu.edu/Virtual/index.php?f=4>

This is a great resource site to lead you to link with other sites. This specific site would be best accessed by a teacher and have him/her link to a more appropriate student oriented program.

Directory of Mathematics Resources

<http://galaxy.einet.net/galaxy/Science/Mathematics.html>

This is also a resource site that will guide you to other sites. The resources are once again best accessed by a teacher who can then allow the student to link to the appropriate sites.

Linda Sheffield's Web Pages

Linda Sheffield is a mathematics educator who has an interest in gifted students. She has a large number of resources listed at her websites:

- Resources for Teachers
<http://www.nku.edu/%7Emathed/tr.html>
- Resources for PreK – grade 12 students
<http://www.nku.edu/%7Emathed/p12sr.html>

Mathematics Competitions

Mathematics competitions are a good way to encourage problem solving among mathematically able students who usually enjoy participating. Sets of old problems are a good resource for competition practice or classroom activities. The three contests listed here are among the most well-known.

American Mathematics Competitions (AMC)

<http://www.unl.edu/amc>

The purpose of the AMC is to increase interest in mathematics and to develop problem solving ability through a friendly competition. The questions range in difficulty from easy to very difficult in order to appeal to a broad range of students. The exam covers material normally associated with the 7th and 8th grade mathematics curriculum including (but not limited to) such topics as the arithmetic of integers, fractions and decimals, percent and proportion, number theory, informal geometry, perimeter, area, volume, probability and statistics, and logical reasoning. Copies of prior year exams are available for purchase.

MATHCOUNTS

<http://mathcounts.org/>

MATHCOUNTS is a national program for competition in mathematics for 7th and 8th graders. School teams use materials provided by MATHCOUNTS to practice throughout the fall. Four students are selected to compete as a team and as individuals in written and oral competitions at a local meet held in February. Winners progress to state and national competitions.

Mathematics Olympiads for Elementary and Middle Schools

<http://www.moems.org/>

These are contest problems that are administered 5 times during the school year within the school. The top few scores are sent as the "team score" as part of the competition. The problems are challenging and engaging. The elementary section is challenging for grades 4-6 but precocious third graders have been known to participate. The middle school level is geared to grades 7 and 8. The most benefit can be obtained from this program if the problems are debriefed in a classroom workshop where strategies are shared. There is a fee for participating

in the competition. Previous problems are available and are an excellent source of rigorous problems for upper elementary students who need a challenge.

Here is a sample Olympiad problem for elementary grades:

$$\begin{array}{r} A H \\ + A \\ \hline H E E \end{array}$$

In the addition problem above, different letters stand for different digits. A H represents a two digit number and H E E represents a three digit number. What number does H E E represent?

Here is a sample problem from the Middle school contest problems:

Of all the mathletes at Smith Middle School, 80% own computers and 40% are in the band. However, 10% of all mathletes neither own their own computers nor are in the band. What percent of the mathletes both own their own computers and are in the band?

Games

Games can provide challenging contexts in which learning takes place. The following are some suggestions.

Krypto

<http://www.mphgames.com/>

This card game for all ages consists of a deck of cards that contains numbers from 1 to 15. A hand of five cards is dealt with a sixth or "target" card. The challenge is to use all five cards and the four basic operations to get the number on the target card. This is an excellent game for developing number sense, mental arithmetic skills, and sharpening computation skills. It can be played as a whole class, small group, or solitaire game.

SET

<http://www.goodnet.com/~setgame/>

This card game requires visual discrimination and reasoning in the identification of sets of cards with various attributes. The one rule of the game is to find three cards where the characteristics of shape, color, number, and shading are either all the same or all different across the three cards. It is challenging for ages 6 to adult. It can be played as a whole class, small group, or solitaire game.

GEMS Math Around the World

<http://www.lhs.berkeley.edu/gems/>

Math Around the World is a unit in the Great Explorations in Math and Science (GEMS) series, which was developed by the Lawrence Hall of Science in Berkeley, CA. While most of the units in this series have science as a primary focus, with mathematics used as needed, *Math Around the World* uses games played in different parts of the world to integrate math and cultural studies. Eight games and puzzles from four continents are presented, including Mancala, NIM, Kalah, and The Tower of Hanoi